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Calculate the solubility S ($\frac{\text{mol}}{\text{L}}$) of CaF_2 in a $Ca(NO_3)_2$-solution 0.100 $\frac{\text{mol}}{\text{L}}$.

$K_{sp_{CaF_2}} = 3.9 \times 10^{-11}$

Solution

$CaF_2(s) \rightleftharpoons Ca^{2+}(aq) + 2\ F^-(aq)$ $\qquad K_{sp_{CaF_2}} = \left[Ca^{2+}\right] \times \left[F^-\right]^2 = 3.9 \times 10^{-11}$

For every CaF_2-particle going into solution, 1 Ca^{2+}-ion and 2 F^--ions are formed.

When dissolving in water, we can conclude that $[Ca^{2+}] = S$ and $[F^-] = 2.S$.

Because we are dissolving CaF_2 in a $Ca(NO_3)_2$-solution, containing Ca^{2+}, the former expression no longer holds: most of the Ca^{2+}–ions were already present (supplied by the $Ca(NO_3)_2$) and only a few were supplied by the dissolving of CaF_2. So we have to calculate the solubility starting from the latter equation: $[F^-] = 2.S$ or $S = \frac{\left[F^-\right]}{2}$.

And since $\left[F^-\right] = \sqrt{\frac{K_{sp_{CaF_2}}}{\left[Ca^{2+}\right]}}$, the solubility in this case is $S = \frac{\sqrt{\frac{K_{sp_{CaF_2}}}{\left[Ca^{2+}\right]}}}{2}$

As we already mentioned, there are two kinds of Ca^{2+}–ions in the saturated solution:

- Ca^{2+}–ions already present before dissolving CaF_2: c_o
- Ca^{2+}–ions supplied by the dissolving of CaF_2 : S

$$S = \frac{\sqrt{\frac{K_{sp_{CaF_2}}}{\left[Ca^{2+}\right]}}}{2} = \frac{\sqrt{\frac{K_{sp_{CaF_2}}}{c_o + S}}}{2}$$

Since $S << c_o$:

$$S = \frac{\sqrt{\frac{K_{sp_{CaF_2}}}{c_o + S}}}{2} = \frac{\sqrt{\frac{K_{sp_{CaF_2}}}{c_o}}}{2} = \frac{\sqrt{\frac{3.9 \times 10^{-11}}{0.100}}}{2} = 9.9 \times 10^{-6}\ \frac{\text{mol}}{\text{L}}$$